

1. A(9,0,4), B(2,3,0)

(d) $\vec{AB} = \langle -7, 3, -4 \rangle$

(e) $\sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(-\frac{2}{3}\right)^2} = \sqrt{\frac{8}{9}} = 2$

3. $\vec{AB} - \vec{BC} = \vec{BA}$

(e)

4. $\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 1 & 1 \\ -1 & 1 & 4 \end{vmatrix} = \begin{vmatrix} (-2-3)i \\ -(-1-6)j \\ +(-1-4)k \end{vmatrix} = \langle -5, 7, -3 \rangle$

5. $C = \sqrt{x^2 + y^2} \Rightarrow x^2 + y^2 = C^2$ circles

(e)

6. $z = mx + ny + c$

(a) $m = \frac{\partial z}{\partial x}|_{(0,0)} = 2e^{x^2}|_{(0,0)} = 2$

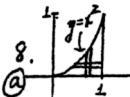
$n = \frac{\partial z}{\partial y}|_{(0,0)} = -\sin(y)|_{(0,0)} = 0$

z = 2x + 0y + c. z = 2 when x = 0, y = 0
⇒ c = 2. z = 2x + 2

7. $|\nabla f(3,2)| = \left| \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \right|_{(3,2)}$

= $\left| \langle 2x^2y, x^2 \rangle \right|_{(3,2)} = \left| \langle 12, 9 \rangle \right|$

= $\sqrt{144+81} = \sqrt{225} = 15$

8.  $\int \int_D \sin(y) dx dy$

9.  $y = \sqrt{r^2 - x^2} \Rightarrow x^2 + y^2 = r^2$

(e) $\int \int_D e^{x^2} r dr d\theta$

10. (d)

11. $\vec{r}(t) = \int \vec{r}'(t) dt = \langle 2e^{2t} + C_1, t^2 + C_2, \sin(t) + C_3 \rangle$

(a) $\vec{r}(0) = \langle 2, 1, 0 \rangle = \langle 2 + C_1, 0 + C_2, 0 + C_3 \rangle$

⇒ $C_1 = 0, C_2 = 1, C_3 = 0$

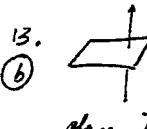
⇒ $\vec{r}(t) = \langle 2e^{2t}, t^2 + 1, \sin(t) \rangle$

⇒ $\vec{r}(0) = \langle 2e^0, 0, \sin(0) \rangle$

12. $\int_C 3y dx + (4x+y) dy =$

(d) $\iint_D \left[\frac{\partial (4x+y)}{\partial x} - \frac{\partial (3y)}{\partial y} \right] dA = \iint_D (4-3) dA$

= $\iint_D 1 dA = \text{area} = \frac{1}{2}(2)(3) = 3$

13.  $\vec{N} = \langle 2, -1, 3 \rangle$

14. $z = y^2$, a parabolic cylinder in
+ direction

15. a helix in the y direction

(e)

16. $\frac{df}{dx}(1,2) = \frac{Af}{Ax} = \frac{f(2,2) - f(0,2)}{2-0} = \frac{7-1}{2} = 3$

17. $\frac{dz}{dt}|_{t=0} = \frac{dz}{dx} \frac{dx}{dt}|_{t=0} + \frac{dz}{dy} \frac{dy}{dt}|_{t=0}$
= $(2x) \cos(4)|_{t=0} + 3y^2 \cdot \frac{dy}{dt}|_{t=0}$
= $0 \cdot 1 + 3(2)^2 \cdot 1 = 12$

18. $\text{div } \vec{F}(P) = 0 + \text{curl } \vec{F}(P) \neq 0$

(a)

19. $\iint_D f dA = [f(4,2) + f(4,0) + f(2,0) + f(2,2)] / (4 \cdot 2)$
= $[3 + 5 + 5 + 6] / (16) = (14)(1/16) = 30/4$

20. $\int_C \vec{v}_f \cdot d\vec{r} = f(3) - f(1) \quad A = \langle x(0), y(0) \rangle$

(a) $B = \langle x(2), y(2) \rangle = (6, 2) \Rightarrow f(6, 2) - f(3, 2) = 5 - 5 = 0$

21. (a) $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos(\theta)$

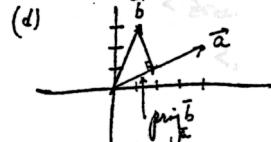
$\langle 4, 2 \rangle \cdot \langle 1, 3 \rangle = \sqrt{17} \sqrt{10} \cos(\theta)$

$10 = \sqrt{20} \sqrt{17} \cos(\theta)$

$\sqrt{17} = \cos(\theta) = \sqrt{2}/\sqrt{17} \Rightarrow \theta = \pi/4$

(b) $\text{comp}_{\vec{a}} \vec{b} = \vec{b} \cdot \frac{\vec{a}}{|\vec{a}|} = \langle 1, 3 \rangle \cdot \frac{\langle 4, 2 \rangle}{\sqrt{20}}$
 $= \frac{10}{\sqrt{20}} = \frac{5}{\sqrt{5}} = \sqrt{5}$

(c) $\text{proj}_{\vec{a}} \vec{b} = \sqrt{5} \frac{\langle 4, 2 \rangle}{\sqrt{20}} = \langle 2, 1 \rangle$



22.  $\vec{PQ} = \langle 1, 3, 3 \rangle$
 $\vec{PR} = \langle 1, 2, 1 \rangle$

$\vec{N} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 1 & 3 & 3 \\ 1 & 2 & 1 \end{vmatrix} = \langle -3, 2, -1 \rangle$

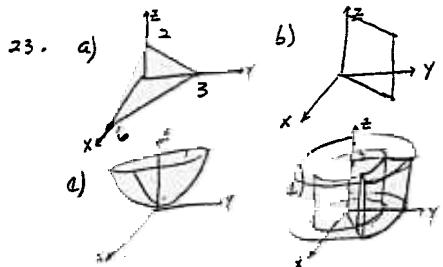
a) $a(x-4) + b(y-3) + c(z-2) = 0$
 $-3(x-1) + 2(y-0) - 1(z-1) = 0$

$+3x - 2y + z = 4$

b) area of $\triangle PQR = \frac{1}{2} |\vec{PQ} \times \vec{PR}|$

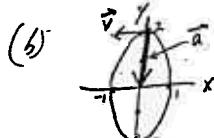
$= \frac{1}{2} \sqrt{9+4+1} = \sqrt{14}/2$

c) $x = 1+t; \quad y = 3t; \quad z = 1+3t$



24. $\vec{r}(t) = \langle \cos(t), 2\sin(t) \rangle$

$$\begin{aligned} \vec{v}(t) &= \langle -\sin(t), 2\cos(t) \rangle \Big|_{t=\frac{\pi}{2}} = \langle -1, 0 \rangle \\ \vec{a}(t) &= \langle -\cos(t), -2\sin(t) \rangle \Big|_{t=\frac{\pi}{2}} = \langle 0, -2 \rangle \\ |\vec{r}(t)| &= \sqrt{(\cos t)^2 + (2\sin t)^2} \Big|_{t=\frac{\pi}{2}} = 1 \end{aligned}$$



$$\begin{aligned} (c) \int_0^{2\pi} |\vec{r}(t)| dt &= \int_0^{2\pi} \sqrt{(\cos t)^2 + (2\sin t)^2} dt \\ &\approx 9.688 \end{aligned}$$

25. (a) $0 = y - \sqrt{x} \Leftrightarrow y = \sqrt{x}$
 $= y - \sqrt{x} \Leftrightarrow y = \sqrt{x+1}$
 $= y - \sqrt{x+2} \Leftrightarrow y = \sqrt{x+3}$



$$(b) \nabla z(1,1) = \left\langle \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right\rangle \Big|_{(1,1)} = \left\langle -\frac{1}{2\sqrt{1}}, 1 \right\rangle \Big|_{(1,1)} = \left\langle -\frac{1}{2}, 1 \right\rangle$$

$$(c) \nabla z(1,1) \cdot \frac{\langle 4,3 \rangle}{5} = \left\langle -\frac{1}{2}, 1 \right\rangle \cdot \left\langle 4,3 \right\rangle = \frac{1}{5}$$

$$(d) \text{not } \langle 2, 1 \rangle, c \neq 0$$

26. $\vec{r}(t) = \langle 0, -3t \rangle, \vec{v}(t) = \langle 0, -3 \rangle$

$$\begin{aligned} \vec{v}(0) &= 50 \langle \cos 60^\circ, \sin 60^\circ \rangle = \langle 25, 25\sqrt{3} \rangle \\ &= \langle 0, 1 \rangle \Rightarrow \vec{v}(t) = \langle 0, -3t + 25\sqrt{3} \rangle \end{aligned}$$

$$\vec{r}(t) = \langle 25t, -16t^2 + 25\sqrt{3}t + c_2 \rangle$$

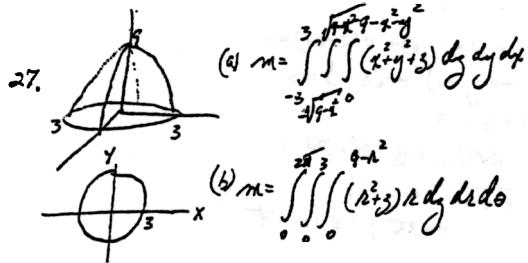
$$\vec{r}(0) = \langle 0, 0 \rangle = \langle 0, c_2 \rangle$$

$$\vec{r}(t) = \left\langle \frac{25t}{\sqrt{16t^2 + 25\sqrt{3}t + c_2}}, -\frac{16t^2 + 25\sqrt{3}t + c_2}{\sqrt{16t^2 + 25\sqrt{3}t + c_2}} \right\rangle$$

$$x(t) = 60 \Leftrightarrow 25t = 60 \Rightarrow t = \frac{12}{5}$$

$$y(t) = -16t^2 + 25\sqrt{3}t \Big|_{t=\frac{12}{5}} = -11.8 \text{ ft}$$

too high \Rightarrow shot misses



28. (a) $\iint_S f(x,y) \, dA$

$$(b) \int_0^1 \int_0^{\sqrt{1-y^2}} g(x,y) \, dy \, dx$$

$$(c) \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{1-y^2}} h(x,y) \, dy \, dx + \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{1-y^2}} h(y,x) \, dy \, dx$$

29. (a) $\int_C \vec{F} \cdot d\vec{r}$ $C: \vec{r}(t) = \langle 1+st, 1+st, st \rangle$

$$dt = \langle 2, 1, 1 \rangle \, dt, \quad t: 0 \rightarrow 1$$

$$= \int_0^1 \langle 1+st, 1+st, st \rangle \cdot \langle 2, 1, 1 \rangle \, dt$$

$$= \int_0^1 (2+8t+3+6t+st+2t) \, dt = \int_0^1 (9+8t) \, dt$$

$$= 9t + 13t^2 \Big|_0^1 = 9+13 = 22$$

(b) No: \vec{F} is not a conservative vector field because $\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 & 1 & 1 \end{vmatrix} = \langle 1, 1, 1 \rangle \neq \vec{0}$.

30. $z = 6 - 2x - 3y \quad \vec{r}(x,y) = \langle x, y, 6-2x-3y \rangle$

$$\iint_S \vec{F} \cdot d\vec{S}, \quad d\vec{S} = \left\langle \frac{\partial \vec{r}}{\partial x}, \frac{\partial \vec{r}}{\partial y} \right\rangle dy \, dx$$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 1 & -3 \end{vmatrix} = \langle 1, 0, -2 \rangle \times \langle 0, 1, -3 \rangle dy \, dx$$

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \langle x, y, 6-2x-3y \rangle \cdot \langle 0, 1, -3 \rangle dy \, dx$$

$$= \iint_D (2x+3y+6-2x-3y) dy \, dx$$

$$= 6 \text{ area of } D = 6 \cdot \frac{6}{2} = 18.$$

31. (a) $\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \bar{\nabla} \cdot \vec{F} \, dV$

where S is the surface enclosing the object E and S is oriented outward

(b) $\iiint_E \bar{\nabla} \cdot \vec{F} \, dV = \iiint_E (6x+3y) \, dV$

$$= 3 \iiint_0^3 \int_0^{\pi} \int_0^{2\sqrt{3}-z^2} (6x+3y) \, dx \, dy \, dz = (3 \text{ volume})$$

$$= 3 \cdot 6 = 18$$